Rewarding Learning

ADVANCED
General Certificate of Education
2022 Reserve Series

Further Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

AFM11

[AFM11]
WEDNESDAY 29 JUNE, AFTERNOON

TIME

2 hours 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all twelve** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all twelve questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Show that for $n \geqslant 1$

$$\sum_{r=n+1}^{2n} r(4r-1)(4r+1) \equiv 60n^4 + 56n^3 + \frac{21}{2}n^2 - \frac{1}{2}n$$ [9]

2 Using mathematical induction, prove that

$$\sum_{r=1}^{n} \frac{1}{4r^2-1} \equiv \frac{n}{2n+1}$$

for all integers $n \geqslant 1$ [8]

3 Evaluate

$$\int_1^{\infty} \left(\frac{1}{x+1} - \frac{1}{x}\right) \mathrm{d}x$$ [5]

4 **Fig. 1** below shows the curve C with polar equation

$$r = 2 + \sin\theta$$

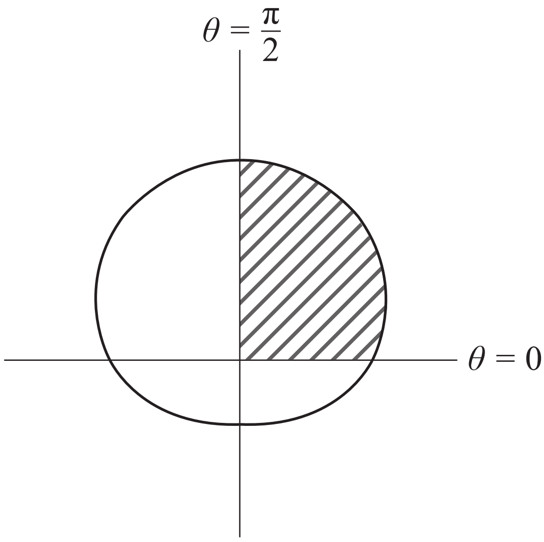

Fig. 1

Find the exact area of the shaded region enclosed between the curve C and the half-lines

$\theta = 0$ and $\theta = \frac{\pi}{2}$ [9]

5 Find the equation, $y = \mathrm{f}(x)$, of the curve which passes through the origin and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \sinh x$$ [10]

6 **(i)** Using partial fractions, show that

$$\frac{7 - x}{(1 + x^2)(1 - 3x)} \equiv \frac{2x + 1}{1 + x^2} + \frac{6}{1 - 3x}$$ [5]

(ii) Hence find a series expansion for

$$\frac{7 - x}{(1 + x^2)(1 - 3x)}$$

up to and including the term in x^4 [7]

(iii) State the range of values of x for which this series is valid. [3]

7 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 16x^2 + 2$$ [13]

8 **(a)** Let $f(x) = (1 + x^2)\sin x$

(i) Show that $f''(x) = \sin x - x^2 \sin x + 4x \cos x$ [4]

(ii) Hence use Maclaurin's theorem to derive the series expansion for $f(x)$ up to and including the term in x^3 [5]

(b) Assuming that the values of x are small, find approximate solutions to the equation

$$30 \sin x + 400 \cos x = 401$$ [5]

9 **(a)** The curve C is given by

$$y = \sin^{-1}\left(\frac{1}{\sqrt{1 + x^2}}\right) + \ln(1 + x^2) \qquad x > 0$$

Show that C has only one stationary point. [9]

(b) Using the substitution $u = x + 1$, or otherwise, find the exact value of

$$\int_{-1}^{2} \frac{dx}{4x^2 + 8x + 13}$$ [9]

10 Let $I_n = \int_0^1 x^n (1-x)^{\frac{1}{2}}\,\mathrm{d}x$ where n is a non-negative integer.

(i) Prove that, for $n \geqslant 1$,

$$I_n = \frac{2n}{2n+3} I_{n-1}$$ [10]

(ii) **Fig. 2** below shows the graph of

$$y = x^2 (1-x)^{\frac{1}{2}} \qquad \text{for } x \geqslant 0$$

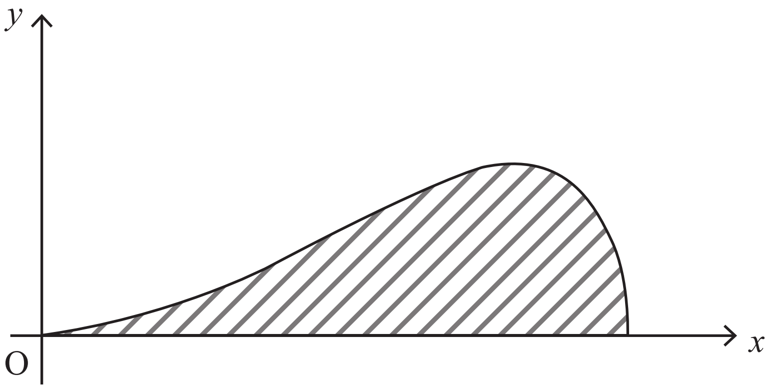

Fig. 2

Find the area of the shaded region enclosed between the curve and the x-axis. [7]

11 Let $y = \cosh^{-1}x$

(i) Prove that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{x^2 - 1}}$$ [5]

(ii) Express $25x^2 + 20x + 3$ in the form $(Ax + B)^2 + C$ [1]

(iii) Hence show that

$$\int \frac{10x + 5}{\sqrt{25x^2 + 20x + 3}}\,\mathrm{d}x = \frac{1}{5}\left[\cosh^{-1}(5x + 2) + 2\sqrt{25x^2 + 20x + 3}\right] + c$$

where $x > -0.2$ [8]

12 Let $z = \cos\theta + \mathrm{i}\sin\theta$

(i) Show that

$$z - \frac{1}{z} = 2\mathrm{i}\sin\theta$$ [3]

(ii) Hence show that

$$\sin^6\theta = \frac{1}{32}(10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta)$$ [9]

(iii) Hence solve the equation

$$2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta = 19 \qquad 0 < \theta < \pi$$ [6]

THIS IS THE END OF THE QUESTION PAPER